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## Abstract

This paper concerns some aspects of uncertainty evaluations in multi-criteria decision-making (MCDM) in the framework of e-commerce website recommendation. The emphasis is on the interest of handling uncertainty with possibility distributions in a MCDM process where evaluations coming from users present variability. Thus, we consider the propagation of possibility distributions through the multi-criteria aggregation made by a Choquet integral that enables to take the interactions between the decision-making criteria into account. To support the recommendation process, location and uncertainty indicators of possibility distributions are defined as well as their marginal contributions to the aggregated result. The proposed approach is applied here to the problem of the choice of an e-commerce website for purchase purposes but it can also be used for dealing with uncertainty in other complex problems.

***Keywords:*** *uncertainty propagation, possibility theory, Choquet integral aggregation, decision-making support system, e-commerce website choice.*

# I. INTRODUCTION

In recent years, there has been a growing interest in the need for designing intelligent systems to address complex problems in engineering, business and social applications. In order to deal with the considered applications, there is often a need to fuse information coming from a variety of sources (physical sensors, measurement systems, experts' opinions) and having different formats (numerical, statistical, linguistic) [1]. Moreover, due to their nature, the sources are different concerning the reliability and uncertainty of the information produced. Therefore, one of the most challenging issue in complex systems is to handle real-world uncertainties effectively. In this line, this paper is a contribution to the estimation and propagation of uncertainty in complex decisional problems. Indeed, the final aim of any measurement process rests on the acquisition of evidence that enables to understand and formulate decisions about some problems [2]. More specifically, the paper addresses the problem of high quality decisions in web-shopping activities and makes propositions to improve the quality of information available to consumers. Indeed, in this emerging domain as well as in many other areas such as medicine, geology, robotics, …, human experts are essential, but they are not able to solve all the numerous related problems. It is therefore desirable to develop computer based systems which incorporate the available knowledge to provide high-level advice to people trying to solve these problems. In this sense, the proposed methodology can be applied to other decision-making situations within a context of justification of decisions with regard to the problem of uncertainty and the risks it involves.

In fact, at the basic level, the assessment and presentation of the effects of uncertainty for complex systems can be viewed in a generic way as the study of functions of the form :

$y=f(\mathbf{m})$

where the function $f$ represents the model, $\mathbf{m}=[m_1,m_2,..,m_n]$ is a vector of basic information inputs and $y$ is the high-level information output.

In practice, $f$ can be quite complex (non linear function, computer program involving complex processing, e.g. fuzzy, neuronal, evolutionary). Here we study the case where $f$ is a Choquet integral. Indeed, in many problems the best known and most extensively used function $f$ is a linear function, e.g. a weighted mean, but it does not allow considering interactions between variables that are often present in applications. Therefore in this paper, to improve the decision-making support, we firstly propose to consider the 2-additive Choquet integral that allows considering mutual interactions for pairs of variables.

Next the goal of an uncertainty analysis is to determine the uncertainty of $y$ that results from the uncertainty in the elements of $\mathbf{m}$ , and further how the uncertainty of $\mathbf{m}$ affects the uncertainty of $y$ through $f$ (sensitivity analysis). To carry out this analysis, the uncertainty in the elements $m_i$ must be characterized. Here, the elements $m_i$ are assumed to be characterized by a possibility distribution $\pi_i$. This uncertainty representation will not be deeply discussed here versus other ones (probability, fuzzy random variables, intervals, … [3][4][5][6][7][8]) but some reasons will be referred to in section IV through our application domain. Here, in the particular application context, the elements of $\mathbf{m}$ are criteria evaluations (within the range [0,20]) provided by users of e-commerce websites, and $y$ is the overall evaluation also within the range [0,20]. Providing the decision-maker with a global uncertain evaluation (e.g. a propagated possibility distribution) guarantees that no information is lost in the information fusion processing. However, it is not necessarily easily understandable. As a decision-making support system is aimed at representing and improving the way people use their reasoning and data processing abilities, significant and relevant indicators for aiding the decision maker must be added. The proposed indicators are based upon descriptive measures on the propagated distribution such as location and uncertainty, which are reinterpreted in the framework of decision making. Indeed, the analysis is viewed under a linear decomposition of the indicators for $y$ into components derived from the indicators of $\mathbf{m}$; the size of these components provides the decision maker indications of importance and variability of the elements of $\mathbf{m}$.

The paper is organised as follows. Section 2 recalls the propagation mechanism of possibility distributions into a 2-additive Choquet integral. Then in section 3, we address the issue of how to support the decision-makers in this MCDM possibility representation of uncertainty. In this view, we propose to describe possibility distributions in terms of interpretable indicators. Only location and uncertainty indicators are considered in this paper. The interpretation in terms of marginal contributions of each elementary indicator on the overall indicators constitutes the basis of proposed recommendation functionalities. In section 4, we illustrate our purpose with e-recommendation applications.

In the remaining of the paper, the following notations will be used:

$S = \{s^1,..,s^l,..,s^q\}$ is the set of q e-commerce websites to be evaluated.

$\mathbf{m}^l = [m_1^l, m_2^l,..,m_n^l]$ the vector of satisfaction degrees of $s^l$ with respect to the $n$ considered elementary criteria.

$y^l$ the overall evaluation $\mathbf{m}^l$.

$\mathrm{CI}(\mathbf{m}^l)$ 2-additive Choquet integral of the elements of $\mathbf{m}^l$ : $y^l = \mathrm{CI}(\mathbf{m}^l)$

$v_i$ coefficient of importance of the criterion $i$ (called also Shapley coefficient)

$I_{ij}$ coefficient of interaction between the criteria $i$ and $j$

$\mathrm{H}(\mathbf{m}^l)$ the simplex domain corresponding to the ranking of the elements of $\mathbf{m}^l$

$\Delta\mu_i^l$ linear coefficient of importance of criterion $i$ in the simplex domain defined by $\mathbf{m}^l$

$\pi_i$ the possibility distribution associated to the evaluation $m_i$ of criterion *i*

$\pi_y$ the possibility distribution associated to the overall evaluation $y$

$\Pi(A)$ the possibility measure of the set $A$ for a possibility distribution $\pi$ : $\Pi(A) = \sup_{x\in A} \pi(x)$

$N(A)$ the necessity measure of the set $A$ for a possibility distribution $\pi$ : $N(A) = 1 - \sup_{x\in \bar{A}} \pi(x)$

$P(A)$ the necessity measure of the set $A$ for a probability distribution $p$ : $P(A) = \int_{x\in A} p(x)dx$

$F$ the cumulative probability function associated to a probability distribution $p$ : $F(x) = \int_{-\infty}^{x} p(x)dx$

$P^*(A)$ the upper probability measure of the set $A$ defined by the possibility measure $\Pi(A)$

$P_*(A)$ the lower probability measure of the set $A$ defined by the necessity measure $N(A)$

$E^*(\pi)$ mean value of the upper probability distribution defined by a possibility distribution $\pi$ : $E^*(\pi) = \int_{-\infty}^{+\infty} xdF_*(x)dx$

$E_*(\pi)$ mean value of the lower probability distribution defined by a possibility distribution $\pi$ : $E_*(\pi) = \int_{-\infty}^{+\infty} xdF^*(x)dx$

$MD(\pi)$ the location indicator of a possibility distribution $\pi : MD(\pi) = (E^*(\pi) + E_*(\pi))/2$

$\Delta(\pi)$ the uncertainty indicator of a possibility distribution $\pi : \Delta(\pi) = E^*(\pi) - E_*(\pi)$

$C(\pi_i)$ contribution of the possibility distribution $\pi_i$ to the overall possibility distribution $\pi_y$

$\pi_y^k$ decomposition part of the overall possibility distribution $\pi_y$ in the simplex domain *k*

$\pi_i^k$ decomposition part of the elementary possibility distribution $\pi_i$ in the simplex domain *k*

$C(\pi_i^k)$ contribution of the possibility distribution $\pi_i^k$ to the possibility distribution $\pi_y^k$

$C\Delta_i$ contribution of the elementary uncertainty indicator $\Delta(\pi_i)$ to the overall uncertainty indicator $\Delta(\pi_y)$

## II. THE 2-ADDITIVE CHOQUET INTEGRAL AGGREGATION OF POSSIBILITY DISTRIBUTIONS

### A. Choquet integral aggregation

Often, the decision-maker problem is to make a trade-off between the different criteria evaluations involved in the considered problem. This leads to consider compromise operators (the aggregated evaluation is between the minimum and the maximum of the elementary evaluations), and thus to disqualify those that model severe or tolerant behavior (such as *t*-norms and *t*-co-norms). Thus, according to these considerations, the operators of the Choquet Integral (*CI)* family [9] are particularly well-adapted because they include a lot of generalized mean operators (*i.e*. those included between the min and the max operators). Moreover, they can be written under the form of a conventional weighted mean modified by effects coming from interactions between elementary evaluations. According to the application context, we will consider here a particular case of Choquet fuzzy integrals known as the 2-additive Choquet integral that considers only interactions by pairs. In the following, the definition of the 2-additive Choquet integral and its principal properties will be briefly recalled.

The expression of the 2-additive Choquet fuzzy integral can be written in the form of Equation 1:

$$CI(\mathbf{m}^l) = \sum_{i=1}^{n} v_i\, m_i^l - \frac{1}{2}\sum_{i>j} I_{ij} \left| m_i^l - m_j^l \right| \qquad (1)$$

This equation involves two types of parameters [10]:

- the weight of each elementary performance expression in relation to all the other contributions to the overall evaluation by the so-called *Shapley coefficients* $v_i$ 's, that satisfy $\sum_{i=1}^{n} v_i = 1$, which is a natural condition for decision-makers,
- the *interaction coefficients* $I_{ij}$ of any pair of performance criteria (i,j), that range within [-1,1]:
  - a positive $I_{ij}$ implies that the criteria are complementary (positive synergy),
  - a negative $I_{ij}$ implies that the criteria are redundant (negative synergy),
  - a null $I_{ij}$ implies that no interaction exists, the criteria are independent; thus $v_i$ *'s* acts as the weights in a common weighted mean (the equation 1 becomes $CI(\mathbf{m}^l) = \sum_{i=1}^{n} v_i\, m_i^l$ ).

An important point is that, the *CI* has a linear form on the simplex domain $H(\mathbf{m}^l)$ corresponding to the ranking defined by the elements of $\mathbf{m}^l$. The *CI* can thus be written as [11]:

$$CI(\mathbf{m}^l) = \sum_{i=1}^{n} \Delta\mu_i^l . m_i^l \qquad (2)$$

with $$\Delta\mu_i^l = v_i + \frac{1}{2}\sum_{j>i} I_{ij} - \frac{1}{2}\sum_{j<i} I_{ij} \qquad (3)$$

This linear expression per simplex domains will be useful for the uncertainty contribution determination (section III).

*B. Propagation of possibility distributions*

As already introduced, we propose to handle uncertainty in a multi-criteria evaluation process. Evaluations are no longer quantitative precise values but fuzzy assessments, i.e. possibility distributions. Computing fuzzy valued Choquet integrals is a topic of wide interest [12][13][14]. The propagation of possibility distributions through the Choquet integral obeys Zadeh's extension principle[1] [15]: $\pi_y(y) = \sup_{(m_1,\ldots,m_n)/_{CI(m_1,\ldots,m_n)=y}} (\min(\pi_1(m_1),\ldots,\pi_n(m_n)))$

where $\pi_y(y)$ is the overall evaluation of a vector **m** and $\pi_i(m_i)$ the elementary possibility distributions. This principle is nothing else than the equivalent of the propagation of probability distributions [3]. But as the axioms of probability and possibility theories are not identical the *sup* operation replaces the *sum* operation and the *min* operation replaces the *product* operation.

Here we limit our study to the case of mono modal piecewise linear distributions: to each criterion *i* a trapezoidal or triangular possibility distribution $\pi_i$ is associated. $\pi_i$ is thus defined by 4 parameters (*a*,*b*,*c*,*d*) with the interval [b,c] as kernel and the interval [a,d] as support. As we are dealing with piecewise linear distributions, and with a piecewise linear propagation function the propagated possibility distribution is also piecewise linear. Moreover, the Choquet integral needs only to be calculated at the intersection points of the ascending and descending parts of the input possibility distributions [12].

This property explains the slope change in the possibility distribution shape such as presented in the following example. Figure 1 illustrates the criteria possibility distributions of four criteria evaluations of a solution $s^l$ and figure 2 the overall possibility distribution obtained by the Choquet integral defined by the coefficients included in table I. The possibility distributions considered in the example lead to two simplex domains. The two sets of linear coefficients are shown table II. The weights of the criteria are quite different in the two domains that reflects the interaction effects.

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[1] No assumption is made about the dependence or independence of the criteria evaluations.

Table I: Choquet integral coefficients

| $\nu_1$ | $\nu_2$ | $\nu_3$ | $\nu_4$ | $I_{12}$ | $I_{13}$ | $I_{14}$ | $I_{24}$ |
|---|---|---|---|---|---|---|---|
| 0.3 | 0.275 | 0.225 | 0.2 | 0.1 | 0.2 | 0.2 | 0.3 |

Table II: Choquet integral linear coefficients for the two simplex domains

| Linear Choquet coeff /domain | $\Delta\mu_i^1$ | $\Delta\mu_i^2$ |
|---|---|---|
| Criterion 1 | 0.05 | 0.25 |
| Criterion 2 | 0.175 | 0.175 |
| Criterion 3 | 0.325 | 0.125 |
| Criterion 4 | 0.45 | 0.45 |

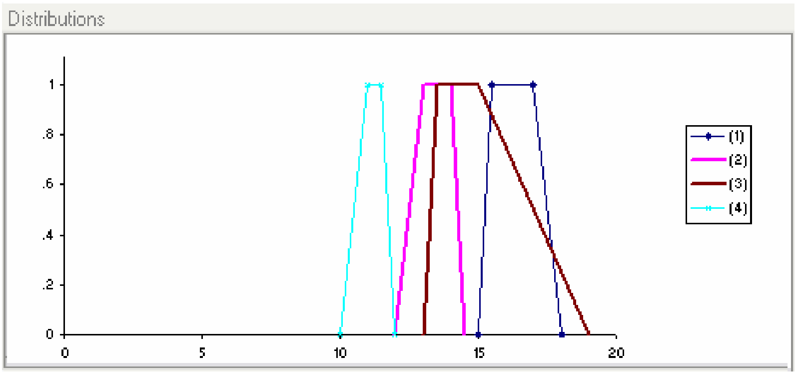


Figure 1: Example of uncertain elementary evaluations

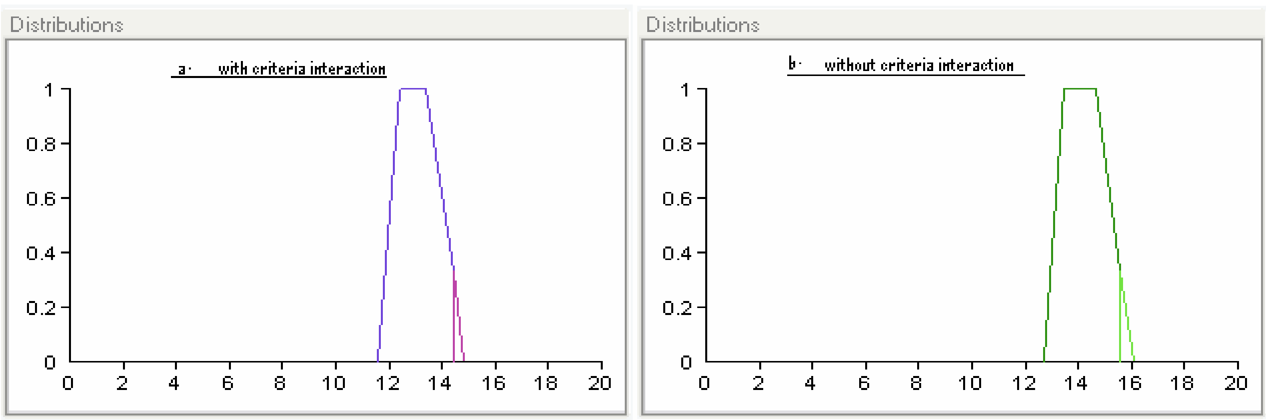


Figure 2a-b: Example of an uncertain overall evaluation

The slope change in the descending part is due to the intersection point at the value 17.66 of the two highest evaluation possibility distributions. As a comparison, the figure 2b gives the aggregated evaluation when the criteria are considered independent, i.e. the interaction coefficients are null. The two kernels are quite different: with criteria interaction we have [12.38, 13.35] and [13.46, 14.62] without interaction.

## III. LOCATION AND UNCERTAINTY INDICATORS FOR DECISION MAKING SUPPORT

### *A. General objectives*

As indicated in the first section, the uncertainty analysis was conducted for the purpose of aiding in a decision. The display of the whole possibility distribution is of value to decision makers. But the decomposition of the resulting overall distribution in terms of contributions of the elementary criteria distributions can provide relevant explanation features for the decision. Indeed, concise quantified interpretable pieces of information reflecting variability of the evaluations are also significant and useful indicators for the decision maker. In this view, we propose, in addition to a location indicator, to associate an uncertainty indicator to each possibility distribution and to consider the propagation of these indicators.

Note that, we are not considering the uncertainty indicator only as a statistical measure, but as a support in the decision making process. Suppose we have a set of ranked solutions described by their overall evaluation distributions. The first ranked solution and the second one are rather close from the point of view of a location indicator (for example, the average). On the other hand, the first one has an important uncertainty whereas the second one has a smaller uncertainty. In some cases, choosing the solution with the least uncertainty could seem more reasonable… This illustrates that the "average" reasoning is not always sufficient and relevant to decide when we are dealing with uncertain evaluations.

Moreover, we want to evaluate the impact of the uncertainty related to each criterion evaluation on the decision making process. The idea is to explain how the uncertainty inherent to a criterion evaluation can contribute to the uncertainty of the overall evaluation (propagated distribution). In other terms, we are evaluating the impact of the uncertainty of each distribution on the decision process and identifying which distribution $\pi_i$ has the most contributed to the uncertainty of the propagated distribution $\pi_y$. The decomposition of the indicators associated to $\pi_y$ into a sum of elementary indicators associated to $\pi_i$ is the fundamental basis of our definition of recommendation functionalities [16] [17].

### *B. Indicator definitions*

Our proposition is based on the fact that a possibility distribution $\pi$ (with $\Pi$ the associated possibility measure and $N$ the associated necessity measure) defines a family of probability distributions $P$ that it dominates, i.e. $\forall A \subset \mathrm{R},\, N(A) \le P(A) \le \Pi(A)$. Two extreme probability distributions of this family are of particular interest, $P_*$ defined by its cumulative probability function $F_* / \forall x \in \mathrm{R}\; F_*(x) = N(]-\infty, x[)$, and $P_*$ defined by its cumulative probability function $F^* / \forall x \in \mathrm{R}\; F^*(x) = \Pi(]-\infty, x])$. Our proposition for the uncertainty indicator is based upon the definition of the upper ($E^*$) and lower ($E_*$) values of the mean value of these probability distributions [18][19]:

$$E^*(\pi) = \int_{-\infty}^{+\infty} x dF_*(x)dx \text{ and } E_*(\pi) = \int_{-\infty}^{+\infty} x dF^*(x)dx \qquad (4)$$

In fact $\left[E_*(\pi), E^*(\pi)\right]$ defines an interval containing all the mean values computed according to all the probability distribution functions *P* dominated by the possibility distribution $\pi$. Moreover they are invariant by linear operations and easily understandable by the users.

We define the uncertainty indicator $\Delta(\pi)$ as the deviation between the upper and the lower values of the mean interval.

$$\Delta(\pi) = E^*(\pi) - E_*(\pi) \qquad (5)$$

Finally, let us denote $MD(\pi) = (E_*(\pi) + E^*(\pi))/2$ the middle of the mean interval of $\pi$. This value allows to reduce the evaluation to one point (in the same way as the average) and thus provides a location indicator consistent with our uncertainty indicator.

*C. Decomposition of the distribution $\pi_y$ into components due to the distributions $\pi_i$*

The idea is to write $\pi_y$ as : $\pi_y(y) = \sum_{i=1}^{n} C(\pi_i)$ (6)

where $C(\pi_i)$ is the marginal contribution of the possibility distribution $\pi_i$ to the propagated distribution $\pi_y$.

As mentioned before, the Choquet integral has a linear expression in simplex domains defined by the ranking of the evaluations. When the evaluations are possibility distributions, the domains are limited by the points of linearity change on the propagated distribution. If we have *p-1* points, we obtain *p* domains. Therefore, the aggregated possibility distribution can be written as the union of the results of the *p* domains thus defined.

$$\pi_y = \bigcup_{k=1..p} \pi_y^k = \bigcup_{k=1..p} \sum_{\mathrm{i=1..n}} \Delta\mu_i^k.\pi_i^k = \bigcup_{k=1..p} \sum_{\mathrm{i=1..n}} C(\pi_i^k) \qquad (7)$$

where $\pi_y^k$ is the part of the propagated distribution corresponding to the k-th domain and $\pi_i^k$ the part of the possibility distribution of the criterion *i* involved in the k-th domain. The latter is obtained by truncating the distribution at the level of the intersection point and by keeping only the increasing part (resp. decreasing part) according to the monotony of $\pi_y^k$ (increasing resp. decreasing). The $C(\pi_i^k)$ 's are possibility distributions that represent the contributions of each criterion *i* to the $\pi_y^k$ in domain *k*. Finally, we obtain the possibility distribution $C(\pi_i) = \bigcup_{k=1..p} C(\pi_i^k) = \max_{k=1..p} C(\pi_i^k)$ .

*D. Decomposition of the uncertainty indicator $\Delta(\pi_y)$ into components due to the uncertainty indicators $\Delta(\pi_i)$*

It is proved in the appendix B that: $\Delta(\pi_y) = \sum_{k=1}^{p} \sum_{i=1}^{n} \Delta\mu_i^k \Delta(\pi_i^k)$ (8)

This equation is the basis for the determination of the impact of the uncertainty of each criterion distribution. Indeed, it allows to identify which one has the most contributed to the uncertainty of the propagated distribution. Thus, the elementary contribution $C\Delta_i$ of each criterion evaluation $\pi_i$ to the uncertainty of $\pi_y$ is defined as:

$$C\Delta_i = \sum_{k=1}^{p} \Delta\mu_i^k . \Delta(\pi_i^k) \quad (9)$$

Let us continue with the preceding example where the aggregated possibility distribution is composed of two domains (see figure 2 and table II). Table III and IV contain the uncertainty indicators and contributions for the two domains (values computed by using equations 8 and 9), and Table IV the overall indicators (values computed by using equation 9).

Table III: Uncertainty indicators and contributions for the simplex Domain1

| ***m***$^l$ | $\Delta_i$ | $C^1\Delta_i$ |
|---|---|---|
| Criterion 1 | 2.194 | 0.109 |
| Criterion 2 | 1.722 | 0.301 |
| Criterion 3 | 3.527 | 1.146 |
| Criterion 4 | 1.222 | 0.549 |
| Propagated Distribution | 2.11 | |

Table IV: Uncertainty indicators and contributions for the simplex Domain2

| **m**$^l$ | $\Delta$ | $C^2\Delta_i$ |
|---|---|---|
| Criterion 1 | 0.055 | 0.013 |
| Criterion 2 | 0.027 | 0.004 |
| Criterion 3 | 0.222 | 0.027 |
| Criterion 4 | 0.027 | 0.012 |
| Propagated Distribution | 0.06 | |

Table V: Uncertainty and contribution indicators for **m**$^l$

| **m**$^l$ | $\Delta$ | $C\Delta_i$ |
|---|---|---|
| Criterion 1 | 2.25 | 0.122 |
| Criterion 2 | 1.75 | 0.305 |
| Criterion 3 | 3.75 | 1.173 |
| Criterion 4 | 1.25 | 0.561 |
| Propagated Distribution | 2.17 | |

Note that due to the small part of the distribution in the domain 2 (see figure 2), the associated uncertainty indicators are small in this particular case. Based on the table V, it can be said that the uncertainty of the propagated distribution is essentially due to the distribution of criterion 3 and at a lower level due to criteria 4 and 2.

## IV. APPLICATION: THE E-RECOMMENDATION PROBLEM

### *A. Introduction*

To illustrate the proposed approach, let us consider the case of the e-recommendation process. Nowadays, because of the tremendous number of e-commerce websites, customers are not only unable to decide which website is the most interesting and the most appropriate for their buying, but also unable to justify on what basis the other e-commerce websites will be rejected. That is why a new generation of websites has emerged: the recommendation websites based on the different customers' evaluations upon these e-commerce websites [20] (see for example: http://www.ciao.fr/shopping_partners). Customers' evaluations are gathered and used to qualitatively evaluate a set of e-business sites with regard to several criteria. For example, for numerical cameras, 115 evaluations are available for Cdiscount.com, 575 for amazon.fr, 46 for Priceminister.com, 6 for expansys.fr, etc. These evaluations result from customers' elementary evaluations with respect to a panel of criteria such as loading speed, site user-friendliness, products variety, price and condition offers, products information, delivery time, after sales service and payment security, etc.

In our study we consider the case of ciao.com (http://www.ciao.fr), which represents a platform of exchanges where members' evaluations are gathered and which proposes a multi-criteria decision making process aimed at supporting a customer in the choice of a suitable e-commerce website (e-retailer) for his/her purchase.

A global evaluation based on the arithmetic mean of the members' evaluations of the e-retailers according to different criteria is currently proposed to customers. But note that the arithmetic mean is an aggregation model that does not take the interactions into account, thus we have replaced it by a 2-additive Choquet integral. Moreover, nothing is proposed in the recommendation about the variability of the evaluations given by the users although this kind of information can represent a great added value in terms of decision-making. Thus, we aim to integrate advanced functionalities into such e-recommendation websites such as: more advanced multi criteria evaluation operators, ranking and comparison of the e-retailer sites, and justification functionalities for choice.

One way to consider variability in customers' evaluations according to each criterion is to construct a probability distribution. But the probability distribution approach is not always adapted. In fact, if we consider the precise evaluation case, building a sound probability distribution is possible only if we have a sufficient number of customers' evaluations (not usually guaranteed in the e-commerce website context). In the imprecise evaluation case, when intervals are considered, they are rarely disjointed or nested in practice; in fact, a more realistic assumption is that all intervals somehow overlap, which prevents computing a probability distribution directly from them.

Considering all these aspects we propose to consider a possibility theory-based uncertainty representation [5][15]. Indeed, a possibility distribution can be useful in any problem where heterogeneous uncertain and imprecise data must be dealt with, e.g. subjective, linguistic-like evaluations and statistical data. The possibility distribution can be built using either a probability-possibility transformation [21] or directly from statistical overlapped intervals [22]. The latter approach is the more appropriate in the context of the application where the customers' evaluations are expressed with stars translated into score-bounded intervals.

In the following, based on the ideas presented in section III, we illustrate the interest of using uncertainty indicators and their contributions to provide the user with recommendations to choose an e-commerce website.

*B. Description of the considered case*

Four criteria (product variety (1), products and services pricing (2), payment security (3), delivery time (4)) have been used to evaluate the e-retailers. The global evaluation of an e-retailer is obtained by the aggregation with a 2-additive Choquet integral modelling the behavior of the e-customers. The parameters of this aggregation operator are shown in the Table VI.

The criteria have a quite similar importance, but there is an interaction between them, especially between products and services pricing and payment security and between product variety and delivery time. Note that the identification of the customers' collective behavior is a problem that is not the object of this paper. The coefficients of table VI have been obtained by using evaluations of customers that have been identified as influencers of the customer population and by applying the learning algorithm of Mori and Murofushi [23]. The possibility distributions corresponding to the evaluations of the different websites according to the different criteria are built with the method described in [22], and are given in Table VII. Evaluations given by users are within the range [0,20].

Table VI: Choquet integral coefficients for the considered application

| $\nu_1$ | $\nu_2$ | $\nu_3$ | $\nu_4$ | $I_{14}$ | $I_{24}$ | $I_{34}$ | $I_{23}$ | $I_{13}$ | $I_{12}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.275 | 0.225 | 0.3 | 0.2 | -0.2 | 0 | -0.1 | 0.3 | 0.2 | 0.1 |

Table VII: Elementary possibility distributions of the different websites

| | Cdiscount.com | Price minister.com | Amazon.com |
|---|---|---|---|
| products variety | (14.5,15,16,16.5) | (15,15.5,17,18) | (15.5,16,17,19) |
| Prices | (17,17.5,18,19) | (12,13,14,14.5) | (12,12.5,13,13.5) |
| payment security | (10,11,12,13) | (13,13.5,15,19) | (15.5,16,17,19) |
| delivery time | (4,6,8,9) | (10,11,11.5,12) | (6,7,9,10) |

The overall possibility distributions for the three sites are shown hereafter.

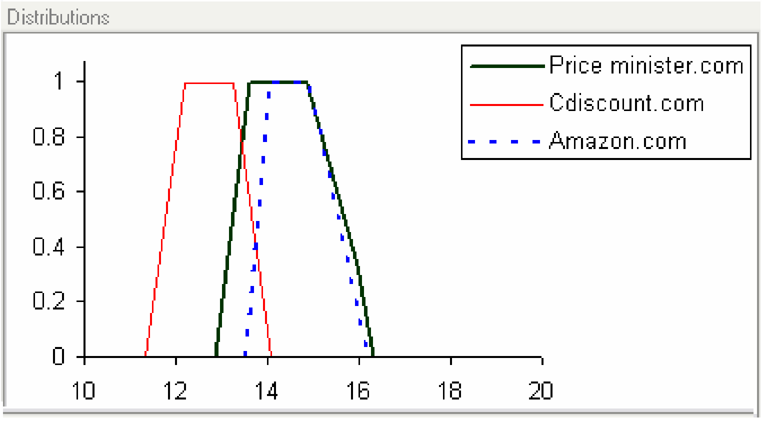


Figure 3: Example of e-retailer overall evaluations

If it is clear that Cdiscount.com is the worst evaluated site, it is more tied for the two others, but Amazon.com is less uncertain. These aspects are quantified in the following using the proposed indicators.

*C. Location and Uncertainty indicators and their contributions*

Tables VIII, IX and X contain the indicators for the three e-retailers considered.

Table VIII: Indicators of Cdiscount.com

| **CDiscount.com** | MD | $\Delta$ | $C\Delta$ |
|---|---|---|---|
| products variety (*Pdtv*) | 15.5 | 1.5 | 0.4875 |
| prices (*Price*) | 17.875 | 1.25 | 0.031 |
| payment security (*Pytsec*) | 11.5 | 2 | 1.2 |
| delivery time (*DTime*) | 6.75 | 3.5 | 0.175 |
| Propagated Distribution ($\pi_{ag}$) | 12.721 | 1.89 | 1.89 |

We have for Cdiscount.com: $C\Delta(Pyt\sec) > C\Delta(Pdtv) > C\Delta(DTime) > C\Delta(\Pr ice)$. Thus customers are aware of possible problems with the payment security. For the website manager of CDiscount.com, one recommendation is to improve services in terms of payment security. The uncertainty of delivery time is higher but has less influence on the overall uncertainty.

Table IX : Indicators of Amazon.com

| **Amazon.com** | MD | $\Delta$ | $C\Delta$ |
|---|---|---|---|
| (*Pdtv*) | 16.875 | 2.25 | 0.506 |
| (*Price*) | 12.75 | 1 | 0.425 |
| (Pytsec) | 16.875 | 2.25 | 0.675 |
| (*DTime*) | 8 | 3 | 0.15 |
| Propagated Distribution | 14.678 | 1.756 | 1.756 |

We have for Amazon.com: $C\Delta(Pyt\sec) > C\Delta(Pdtv) > C\Delta(\Pr ice) > C\Delta(DTime)$. Note that once the criteria payment security and product variety have the same evaluations, they have different uncertainty contributions due to their varying importance in the aggregation.

Table X : Indicators of Priceminister.com

| **Priceminister.com** | MD | $\Delta$ | $C\Delta$ |
|---|---|---|---|
| $(Pdtv)$ | 16.375 | 2.25 | 0.517 |
| $(Price)$ | 13.375 | 1.75 | 0.743 |
| (Pytsec) | 15.125 | 3.75 | 1.08 |
| $(DTime)$ | 11.125 | 1.25 | 0.0625 |
| Propagated Distribution | 14.445 | 2.404 | |

To have a global ranking of these different websites, one issue is to order them according to their location values ($MD$). Thus, we have: Amazon.com $\succ$ Priceminister.com $\succ$ CDiscount.com (where $\succ$ is a ranking operator). Moreover, if we look at their uncertainty indicator, in this particular case we find the same ranking. Thus the recommendation to the customer is to choose the Amazon.com website for his purchase, even though the delivery time is bad for this site. Thus, if the customer's requirements regarding delivery time are higher than the population average, he/she is advised to select Price minister. In fact, all these tables allow customers to adjust their choices according to the additional pieces of information they contain.

## V. CONCLUSION

This paper concerns some aspects of uncertainty in a multi-criteria decision making process. More specifically, we have emphasized the interest of considering possibility distributions instead of precise quantitative evaluations. A method for propagating these possibility distributions using generalized weighted mean aggregation operators such as the Choquet Integral has been exposed. It allows to take interactions into account. In addition, we have proposed to associate uncertainty indicators to each distribution in order to give the user an idea about the variability of the evaluations of other people. Furthermore, these indicators allow to evaluate the impact of the uncertainty associated with each criterion on the decision making process and to explain how the uncertainty inherent to a criterion can contribute to the final result. For the sake of illustration we have considered a process aimed at supporting a customer in the choice of a suitable e-retailer for his/her purchase, but the methodology can also be used for dealing with uncertainty in other complex systems. For the time being, we have only considered mono modal possibility distributions. Work is now in progress concerning the case of bimodal possibility distributions, which will allow to consider controversial aspects of customers' evaluations.

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APPENDIX A

*Proposition 1*: Let us consider a piecewise linear possibility distribution π. This distribution can thus be written as $\pi = \bigcup_{k=1..p} \pi^k$ where $\pi^k$ are linear adjacent possibility distributions (not necessarily normalized); then $\Delta(\pi) = \sum_{k=1}^{p} \Delta(\pi^k)$.

*Proof*: Let us consider a generic distribution $\pi$ plotted in figure 4.

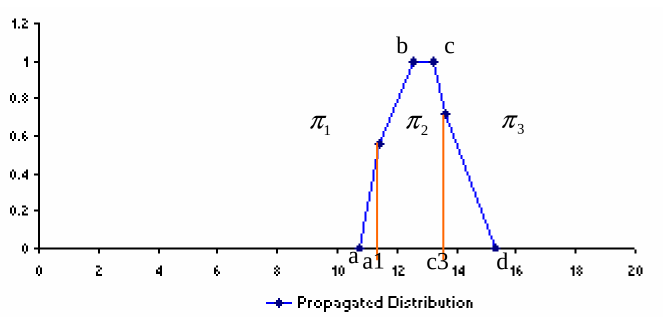


Figure 4: Example of possibility distribution decomposition

Let us first define $E^{*1}$ and $E_{*1}$ for $\pi^1$ that is a non normalized possibility distribution. In this case, we take for $dF_{*1}$ the following degenerated probability distribution that is the weighted sum of a uniform probability distribution and a Dirac one.

$dF_{*1}(x) = \pi(a1).U_{[a,a1]}(x) + (1-\pi(a1)).\delta(x-a1)$ Thus $E_{*1} = \pi(a1).(a1+a)/2+(1-\pi(a1)).a1$

Obviously, $E^{*1}$=a1. With the same reasoning, we have for $\pi^3$: $E_{*3}$=c3 and $E^{*3}$= π(c3).(c3+d)/2+(1-π(c3)).c3

For the distribution $\pi^2$, we have:

$$dF_{*2}(x) = (1-\pi(a1)).U_{[a1,b]}(x) + \pi(a1).\delta(x-a1) \quad dF^{*2}(x) = (1-\pi(c3)).U_{[c,c3]}(x) + \pi(c3).\delta(x-c3)$$

Therefore: $E_{*2}$= (1- π(a1)).(a1+b)/2+π(a1).a1 and $E^{*2}$= (1- π(c3)).(c+c3)/2+π(c3).c3

For the whole distribution π applying the definitions of $E^*$ and $E_*$ leads to:

$E_*$= π(a1). (a+a1)/2+ (1-π(a1)). (a1+b)/2

$E^*$= (1-π(c3)). (c+c3)/2+ π(c3). (c3+d)/2

Thus, $E_* = E_{*1} - (1-\pi(a1)).a1 + E_{*2} - \pi(a1).a1$ that gives $E_* = E_{*1} - a1 + E_{*2} = E_{*1} - E^{*1} + E_{*2}$

In other respects: $E^* = E^{*2} - \pi(c3).c3 + E^{*3} - (1-\pi(c3)).c3$ that gives $E^* = E^{*2} + E^{*3} - c3 = E^{*2} + E^{*3} - E_{*3}$

In conclusion, we have:

$E^* - E_* = E^{*1} - E_{*1} + E^{*2} - E_{*2} + E^{*3} - E_{*3}$ and finally, $\Delta(\pi) = \Delta(\pi^1) + \Delta(\pi^2) + \Delta(\pi^3)$ .

This result can be easily generalized for distributions having any number of linear pieces.

## APPENDIX B

*Proposition 2*: Let us consider the propagated possibility distribution $\pi_y$ and its associated uncertainty indicator $\Delta(\pi_y)$ ; then $\Delta(\pi_y) = \sum_{k=1}^{p}\sum_{i=1}^{n} \Delta\mu_i^k \Delta(\pi_i^k)$ .

*Proof*: from proposition 1 of Appendix A we have: $\Delta(\pi_y) = \sum_{k=1}^{p} \Delta(\pi_y^k)$

Then, due to the linearity in each simplex domain, the uncertainty indicator of the partial propagated distribution $\pi_y^k$ is then: $\Delta(\pi_y^k) = \sum_{i=1}^{n} \Delta\mu_i^k \Delta(\pi_i^k)$

Therefore the following equation holds:

$$\Delta(\pi_y) = \sum_{k=1}^{p}\sum_{i=1}^{n} \Delta\mu_i^k \Delta(\pi_i^k)$$

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