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**UNEDITED ROUGH DRAFT TRANSLATION****: 270076**

BOILING HEAT TRANSFER

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English Pages: 31

Source: A Paper for Presentation at the  
Heat Transfer Conference, June  
5-10, 1961, at Minsk, USSR  
pp. 1-39

SC-957

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S. S. Kutateladze

# BOILING HEAT TRANSFER

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## Introduction

Boiling heat transfer plays a very important part in the solution of a number of problems in new technology and has great application to industrial power engineering. However, the forcing levels of the heat-transfer surfaces in these cases can differ by several orders of magnitude. For example, in refrigeration engineering, when freon boils, the heat flux has densities of the order of  $10^3$  kcal/m<sup>2</sup>hr; in the evaporators of hydroelectric stations they are of the order of  $10^4$ ; in water-moderated water-cooled reactors of nuclear power plants they are of the order of  $10^6$ ; in the elements of jet engines they are of the order of  $10^7$  kcal/m<sup>2</sup>hr.

In nuclear reactors and other highly-forced installations the decisive factor is the critical density of the heat flux; this determines the upper safety limit for the aggregate.

In the apparatus used in industrial power engineering the absolute values of the heat-transfer coefficients are of importance, since in such devices these coefficients are usually of the same

order for warming and for heated media.

Film boiling occurs in hardening processes, at near-critical pressures, and in certain other specific cases.

This report is devoted to the problems of heat transfer in nucleate boiling. Nucleate boiling is characterized by the formation of vapor at individual sites on the heating surface, the so-called evaporation centers, and imparts to the liquid and vapor mixture a very complex inhomogeneous structure. This phenomenon pertains to one of the most complex problems in the hydrodynamics of liquid-vapor systems, namely to flows in which the phase components of the flux are separated into individual formations bounded by the mobile phase interfaces. These formations (bubbles, drops, films), variable in space and time, are very numerous, so that here the probability laws of a multi-element system should be in effect.

At present there are no consistent analytical methods for such systems. The purpose of the present work is to discuss some of the main questions of the boiling heat-transfer theory and to generalize the results of experimental work.

#### Nomenclature

C	-specific heat
D = 2R	-diameter
g	-acceleration due to gravity
$\Delta i$	-difference between the mean enthalpy of the flux and the enthalpy of the saturated liquid
L	-length
p	-pressure
q	-heat-flux density

$q_{cr}$	-critical heat-flux density
$r$	-latent heat of evaporation
$T$	-temperature
$T''$	-saturation temperature above a plane
$w_0$	-circulation rate
$x$	-vapor content of the flux by weight
$\alpha$	-heat-transfer coefficient
$\beta$	-vapor content of the flux by volume
$\gamma$	-specific weight
$\delta$	-transverse linear dimension of the channel
$\$$	-heating of the flow core to below the saturation temperature
$\lambda$	-coefficient of thermal conductivity
$\nu$	-coefficient of kinematic viscosity
$\sigma$	-coefficient of surface tension
$\tau$	-time

Indexes:    "-vapor phase, i-phase interface.

### The Origin of Steam Bubbles

The intensity of nucleate-boiling heat transfer is due to the interaction of convection associated with the directed motion of the entire flow of liquid, and local convection, caused by the origination, growth, and flotation of steam bubbles. With sufficiently high heat-flux densities, i.e., at considerable vapor formation rates, the last process is decisive.

The probability of the origination of a vapor-phase nucleus in a superheated liquid is proportional to the number  $\underline{e}$  raised to a power which is inversely proportional to  $(\Delta t)^2$ . Consequently, vapor bubbles should occur in the immediate vicinity of the heating surface,

where superheating of the liquid is greatest, compared with the saturation temperature above a plane.

Certain works on boiling heat transfer have used G. N. Kruzhilin's erroneous viewpoint [21, 22] that vapor bubbles originate on rough projections. At present we can state that the boiling centers on a solid surface are depressions, pores, concavities at the bases of tubercles, and rough places. Andreyev [5], examining cylindrical pores, was first to indicate such a mechanism. This was then examined by Nesis [33] and Courty and Foust [57]. The origination of a vapor bubble on a conical pore was examined by Courty [57], Bankoff [54], and others. Zuber [73] has published a review of these works.

Thus, the boiling process is due not only to the probability of nucleation with given superheating of the liquid, but also to the probability of the distribution of vapor-formation centers on the heating surfaces. The probable occurrence of vapor bubbles on a solid surface is clearly proved by the frequency distribution curve of their formation; these were obtained for several pressures by Zysina and the author [15]. Analogous curves were later obtained by Treshchev [48] for boiling when the liquid flow core is heated to the saturation temperature. The detachment diameter of bubbles is also a variable value, subject to a certain distribution. Under static conditions, the detachment diameter of a bubble on a smooth surface is defined, according to calculations by Fritz [60], by the formula [23]

$$D_0 = 0,02 \theta \sqrt{\frac{\sigma}{\rho_l - \rho_v}} \quad (1)$$

The variability of  $D_0$  indicates that the value of the edge angle  $\theta^\circ$  cannot be regarded as a definite characteristic of a great expanse of the heating surface. In addition, recent motion pictures made at



the Thermophysics Institute show that with intense boiling, the bubble before detachment has complex form and is attached, in part, to the heating surface by a cylindrical neck.

These facts clearly demonstrate that the hydrodynamic and thermal processes during nucleate boiling can be described only statistically.

### The Rate of Growth of a Vapor Bubble

The rate of growth of a vapor bubble is associated with the heat-flux density at its surface by the equation

$$\frac{dR}{dt} = \frac{q_1}{\rho_1 r''} \quad (2)$$

Boshjakowic [55] used this relation to determine the value  $q_1$  from the experimental data of Jakob et al. [64, 65].

If we consider the liquid medium around the bubble to be immobile,

$$q_1 = C(T - T_1'') \sqrt{\frac{\lambda c r'}{\tau}} \quad (3)$$

where the proportionality factor  $C$ , according to Plesset and Zwick [67], is  $\sqrt{3}$ , while according to Forster and Zuber [59] it is  $\frac{\pi}{2}$ , i.e.,  $C \approx 1.6$ . Here  $T$  is the temperature of the superheated liquid and  $T_1''$  is the saturation temperature at the boundary of the bubble.

According to this formula, for the conditions of the formation of one of the vapor bubbles photographed by Zysina and me, we have

$\tau$ , sec	0.00475	0.0095	0.0285
$q_1$ , kcal/m <sup>2</sup> hr	203,000	143,000	83,000

Under conditions of thermodynamic equilibrium, for a spherical bubble

$$T_1'' = T'' + \frac{2\sigma r}{R(r-r'')} \cdot \frac{dT''}{dP}. \quad (4)$$

During heat transfer, thermodynamic equilibrium is absent, the bubble grows due to evaporation of the liquid in it, and the value  $T_1$  differs from (4). Assuming that there is only vapor in the bubble we can, in the first approximation, relate the evaporation rate and the pressure drop in the vapor boundary layer by the familiar kinetics formula

$$q_1 = \xi r \Delta p \sqrt{\frac{g_m}{2\pi B T''}}, \quad (5)$$

where  $\xi < 1$  is the accommodation coefficient, B is the universal gas constant, and  $m$  is the molecular weight.

The corresponding change in the saturation temperature

$$\Delta T_q = \frac{q_1}{\xi r} \sqrt{\frac{2\pi B T''}{g_m}} \cdot \frac{dT''}{dP}. \quad (6)$$

Adding (4) and (6), we find that

$$T_1'' = T'' + \left[ \frac{2\sigma r}{R(r-r'')} + \frac{q_1}{\xi r} \sqrt{\frac{2\pi B T''}{g_m}} \right] \frac{dT''}{dP}, \quad (7)$$

where  $\frac{dT}{dP} = \frac{A T''(r-r'')}{r r''}$ . In technical units,  $A = \frac{1}{427}$ .

Figure 1 compares the terms in brackets in Formula (7). From the figure it is clearly evident that even when  $\xi = 1$  the correction associated with evaporation kinetics is not only commensurate with the correction for the curvature of the phase interface, but in a number of cases can even exceed it.

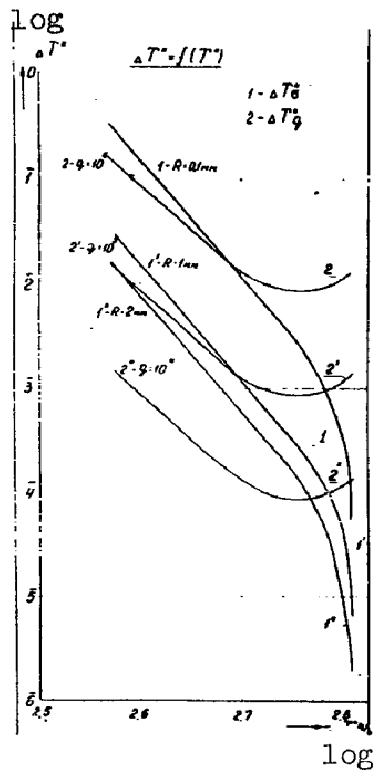


Fig. 1. Values of  $\Delta T''_{\sigma}$  and  $\Delta T''_q$  according to Formula (7).

First Heat Flux Critical Density with Flow in Tubes and Channels

Any examination of the boiling process must begin with the problem of free convection in a large volume of liquid. However, in technology the most essential problem is the forced flow of a boiling liquid in tubes and channels.

At the present time much experimental data are available on critical heat fluxes in tubes, while there is not much available on fluxes in slit channels.

It is extremely important to explain the effect of flow-rate pulsations on the value  $q_{cr}$ . There are a great many operational and thermophysical parameters which determine the limiting density of the heat flux for which nucleate boiling and, consequently, high heat-transfer intensity are still maintained. In this regard, the

question of methods of a generalized analysis of the examined phenomenon and rational forms of the connection between its determining values is of primary importance.

Two hypotheses have been advanced with regard to the boiling crisis in large volumes of liquids.

The first of these, proposed by the author [24, 25], presumes that the transition from nucleate to film boiling is a qualitatively specific process, the basis of which is the loss of stability of the structure of a two-phase boundary layer which existed prior to the crisis.

In this case, as for any hydrodynamic crisis, the transition from one boiling point to another is determined not by a strictly fixed value (in this case  $q_{cr}$ ), but only by its most probable value.

The second hypothesis, most clearly formulated by G. N. Kruzhilin [21] is based on the assumption that during the first boiling crisis no new qualitative peculiarities arise and, consequently, the critical heat flux density is determined by the same initial group of generalized parameters (similarity criteria) as is the heat-transfer coefficient during nucleate boiling. This hypothesis has not been developed for forced flow, while for free convection boiling, as has been pointed out [28], it has resulted in an unjustifiably complex empirical formula.

According to the hydrodynamic hypothesis, the transition from nucleate to film boiling begins with a loss in the stability of the two-phase boundary layer in one of the small regions of the heating surface, and the formation of a stable vapor "spot." The increase in temperature occurring beneath this "spot," due to a decrease in the heat-transfer coefficient, results in further development and

propagation of film boiling along the heating surface. Such a system is in complete agreement with that observed during boiling on heat-resistant (e.g., graphite) rods and plates.

It is not too difficult to describe the conditions of the mechanical interaction between vapor and a liquid for a differential element of the phase interface of such a system. Here, since it is a question of the local stability of structure, the matter of boundary conditions for the entire system does not arise. Accordingly, the system of equations consists of motion and continuity for each phase, and conditions of the mechanical phase interaction at the interface [25].

Treating an inviscid liquid, we can also use Rayleigh's theory of the stability of a double-layer flux [39].

When the problem is treated thus, the value to be determined is the critical vapor rate, and the question arises as to the connection between the local value and the mean heat flux density at the heating surface.

Evidently, the only actual measure of the rate of vapor formation is the value

$$\frac{q}{2r^2}, \quad (8)$$

the maximum amount of vapor which can form per unit of the heating surface per unit time.

Here we will examine the maximum amount of vapor, since in the general case part of the heat flux  $q$  may not be expended on vapor formation (e.g., in a flux with a core which is heated to below the saturation temperature).

Yet another characteristic is evident from the heat balance of the mixture; this can be represented as the criterion

$$K = \frac{r}{\Delta i} , \quad (9)$$

where  $\Delta i$  is the difference between the mean enthalpy of the flux and the enthalpy of the saturated liquid.

In a flow of saturated liquid,  $\frac{1}{K} = x$ , where  $x$  is the vapor content by weight, i.e., in this case the criterion  $K$  actually has purely hydrodynamic significance.

When  $\bar{T} < T''$ , the criterion  $K = \frac{r}{C\delta}$ , where  $\delta = T'' - \bar{T}$  is the subheating of the flow core to the saturation temperature. In this case,  $K$  is purely hydrodynamic only when the heat-transfer mechanism is completely turbulent; this is a measure of the decrease in vapor flux due to condensation in the cold core. If the molecular thermal conductivity is significant, the criterion  $K$  is associated with the Prandtl number  $Pr$  as a thermal criterion. We will not discuss this at this time.

The stability of a two-phase system is always influenced by the kinetic head of the originating phase, the measure of which, in this case, is the value

$$\left(\frac{q}{2r''}\right)^2 \frac{\sigma}{q} , \quad (10)$$

gravity, and surface tension. As the measure of the joint action of these two latter forces (keeping (1) in mind) we can use the value

$$(\delta - \delta'') \sqrt{\frac{\sigma}{\delta - \delta''}} . \quad (11)$$

The square root of the ratio of these values is the criterion [24, 25]

$$k = \frac{q}{2\sqrt{q}r'' \sqrt{\sigma(\delta - \delta'')}} . \quad (12)$$

When  $q = q_{cr}$  criterion (12) becomes definite.

If we use the Rayleigh stability criterion, criterion K assumes the form

$$k = \frac{2cr}{2\sqrt{g\sigma''(1+\frac{r''}{r})} \sqrt{\sigma(r-\sigma'')}} \quad (13)$$

The value of this group for the first critical heat-flux density (the occurrence of film boiling) is designated by the index 1; for the second critical heat-flux density (destruction of the vapor film) the value is designated by the index 2.

The difference between expressions (12) and (13), the value  $\sqrt{1 + \frac{\gamma''}{\gamma}}$ , has no essential significance, and is associated with the solution of the initial system of equations used by Rayleigh.

Various authors [24, 25, 26] have shown that within the framework of these assumptions we have the following connection between the dimensionless criteria:

$$k = \Phi \left[ w_0 \left( \frac{r-\sigma''}{g\sigma} \right)^{1/4}; \frac{\nu^2}{g} \left( \frac{r-\sigma''}{\sigma} \right)^{3/2}; \frac{r}{\sigma^2}; \delta \left( \frac{r-\sigma''}{\sigma} \right)^{1/2} \frac{L}{\delta}; \frac{\Delta t}{T} \right] \quad (14)$$

Here we should give a distribution law for the feed of heat along the heating surface.

For an underheated liquid the hydrodynamic hypothesis has resulted in a conclusion as to the existence (at least in first approximation) of a linear relationship between the criteria K and  $\frac{\Delta t}{r}$  in the form

$$\frac{q_{cr,0}}{q_{cr,\sigma}} = 1 - B \left( \frac{r}{r''} \right)^n \frac{\Delta t}{T} \quad (15)$$

For purely turbulent transfer the coefficient B is a certain constant. If molecular heat conductivity influences the heat transfer near the nascent elements of the vapor phase [e.g., according to (3)], the coefficient B is a function of the Ar and Pr criteria.

Turning to experimental data, let us note that despite numerous investigations, the various results are not in satisfactory agree-

ment. Here we are speaking not only of the purely quantitative discrepancies which can be explained by the instability of the data on critical regimes.

A number of the discrepancies were explained after the role of preincluded volumes in the occurrence of pulsation and the influence of the length of the heated segment had been explained [4, 11, 46].

Most experiments on slit channels and tubes indicate that the influence of the slit-width or the diameter is slight, when

$$\delta > (1 - 3) \sqrt{\frac{\sigma}{\rho - \rho''}}. \quad (16)$$

The linear relationship between  $q_{cr}$  and  $\Delta i$  in the underheating region  $\psi > 0$  is satisfactorily verified by experimental data for relatively open channels with pressures up to critical. Povarin and Semenov have verified the satisfactory nature of this relationship for liquid flow-rates to 40 m/sec [36].

The experiments by various authors [12, 13, 17, 46, 52] have distinctly verified the general nature of relation (15). However, the numerical values of B and  $\underline{n}$  are not the same in various experiments.

The experiments of Zenkevich [13] and Miropol'skiy and Shitsman [17] give a clear relationship between  $q_{cr}$  and  $w_0$  when  $\psi \geq 0$ ; this is satisfactorily described by the empirical formula [24, 25]

$$K_{10} = 0,023 w_0^{1,5} \left( \frac{\delta - \delta''}{g^2 \sigma} \right)^{0,125}. \quad (17)$$

where  $K_{10}$  is the value of the complex  $K_1$  when  $\psi = 0$ .

The experiments of Chirkin and Yukin, Averin and Kruzhilin, and the author, on the flow of water in wide annular channels are also satisfactorily described by a formula of the type in (17) but with a proportionality factor of 0.085 [25, 26]. Ornatskiy's



results [35] are, quantitatively, about 30% lower.

Experiments by Doroshchikin and Frid [11, 17] have shown the weaker effect of liquid flow-rate on  $q_{cr}$ , while there is a slight influence of  $w_0$  on the coefficient B.

The experiments by Alad'yev and Dadonov [3] differ in this respect from the data examined above. In these experiments  $w_0$  has practically no effect on  $q_{cr}$  at low pressures, while at high pressures this effect is relatively slight and the monotonic relation between the coefficient  $B(\frac{\gamma}{\gamma_0})^n$  and pressure is destroyed.

The work by Povarnin and Semenov [36] contains a discussion of the connection between  $q_{cr}$  and  $w_0$  for high liquid flow-rates.

In the region  $x > 0$ , i.e., in the flow of a saturated vapor-liquid mixture, the relationships for  $q_{cr}$  are even more complex. It has been reliably established that the degree of influence of vapor content depends on the mass flow-rate of the mixture or, in other words, on the so-called circulation rate

$$w_0 = \frac{G_{sm}}{\delta \Omega}, \quad (18)$$

where  $G_{sm}$  is the mass flow-rate of the mixture per second, and  $\Omega$  is the cross section area of the canal.

In the absence of pulsations  $(\frac{\partial q_{cr}}{\partial x})_{w_0} < 0$ , i.e., the critical heat flux density decreases, despite the essential increase in the mean flow-rate of the liquid with increased vapor content of the mixture. Comparing this result with (17), we must conclude that increased vapor content of the flux essentially changes the vapor content of the near-wall layer and facilitates destruction of the liquid structures existing in this layer.

From the available experimental data we can note a more or less clear dependence only for the region of vapor content by volume

$\beta < 0.9$ . Since the vapor contents by volume and by weight are associated by the formula

$$\frac{\beta}{1-\beta} \cdot \frac{\sigma''}{\sigma} = \frac{x}{1-x}, \quad (19)$$

the values  $\beta < 0.9$  at low pressures correspond to very slight vapor content by weight, while at high pressures the vapor content by weight is of the order of 0.1 and greater.

R. A. Rybin has processed the experimental data of Alad'yev, Bukhberg, Doroshchuk, Dzhekeg, Layne, Miropol'skiy, Subbotin, et al., pertaining to tubes of 9mm and more diameter, the vapor content  $0 < \beta < 0.9$ , and inlet conditions where the water entering the experimental sector was heated to just below the saturation temperature. The results for two values of the criterion

$$F_* = w_0 \left( \frac{\sigma - \sigma''}{g^2 \sigma} \right)^{1/4} \quad (20)$$

are shown in Fig. 2, from which can be seen the absolute discrepancies among various observations, and the over-all scatter of the experimental points.

It is interesting to note that the average lines at point  $\beta = 0$  practically coincide with calculations by Formula (17).

Figure 3 shows averaged straight lines for various values of criterion (20). As can be seen, as this criterion decreases, i.e., the circulation rate decreases, the relative vapor content also decreases.

For the region of high vapor content by weight ( $\beta > 0.9$ ) there are not enough available experimental data for generalizations.

Thus, systematic compilation and further accumulation of experimental data is a necessary condition for constructing reliable computational dependences for  $q_{cr}$  with flow in tubes and channels.

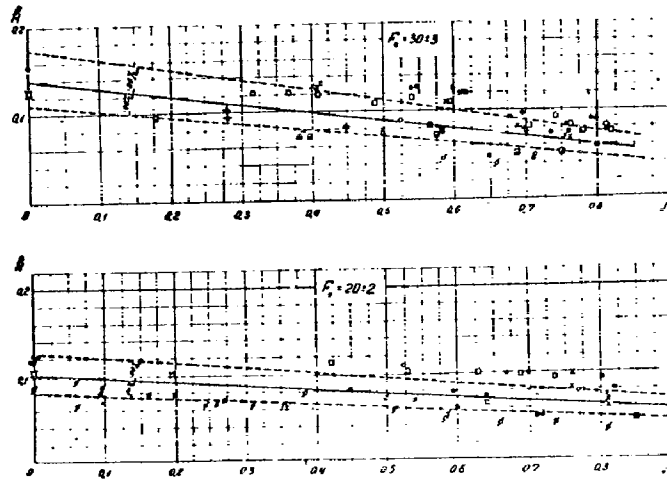


Fig. 2. The value  $K_1$  vs.  $\beta$  and  $F_*$  for flow in tubes.  $\beta < 0.9$ .

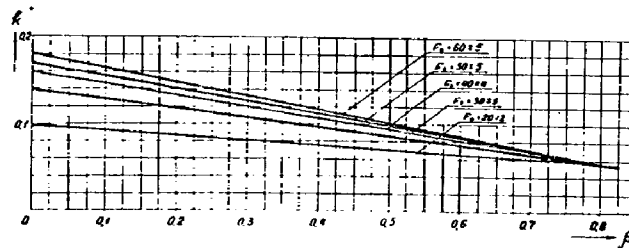


Fig. 3. The value  $K_1$  vs.  $\beta$  and  $F_*$  according to averaged lines.

In this regard, the complex empirical formulas proposed by Zenkevich and Ivashkevich [13, 16] can be considered as a first, and not very successful, attempt.

### Critical Heat-Flux Density with Free Convection in a Large-Volume Liquid

The results of applying the hydrodynamic theory of boiling crises to conditions of free convection in a large-volume liquid are quite familiar, and reduce to the following basic relationships [24, 25]:

$$K_{10} = \text{const}; \quad (21)$$

$$\frac{q_{cr, 2}}{q_{cr, 1}} < 1. \quad (22)$$

Formula (15) expresses the effect of not heating the liquid mass to the saturation temperature.

Using the Rayleigh theory of the stability of a two-layer flux, Zuber and Tribus [72] found that in an inviscid liquid

$$\left. \begin{aligned} 0,12 < K_{20} < 0,16 \\ B = 5,3 (Pr^2 Ar_*)^{-1/4} \\ n = \frac{3}{4} \end{aligned} \right\} \quad (23)$$

Relation (3) was also used in calculating the latter two coefficients.

It is interesting to note that complex B, according to (23), is a very slowly changing value. The theoretical values of (23) closely agree with most experimental data.

However, it is important not to be concerned with this good qualitative and quantitative verification of the theory, but to discuss the various discrepancies noted. Evidently, if these deviations are not associated with inaccurate experiments and data on the physical properties of the investigated substance, they should serve as the starting point for further development of the theory. We are speaking of small ( $< 0.1$ ) values of  $K_1$  for molten salts using approximately the same values for  $K_2$  as in Robin's experiments [38], and deviations of B and  $\underline{n}$  in experiments for biphenyl from the values of these factors for water and alcohol [27].

Very interesting results on critical heat fluxes in mixtures have been obtained by van Wijk, van Stralen [70], and Fastovskiy and Artym [51] for thin-wire boiling. They found that when a binary

mixture has a specific composition, the value  $q_{cr, 1}$  considerably exceeds the corresponding values for each of the components. For pure components the obtained values for the critical heat flux are in good agreement with Formula (21), when Borishanskiy's corrections for viscosity [6, 25] are introduced:

$$K_{20} = 0,13 + 4Ar_*^{-0,4} \quad (24)$$

The mechanism of this phenomenon is far from clear. However, the experiments conducted by Bobrovich et al., at the Thermophysics Institute, using heating surfaces larger than those in the above-mentioned experiments, did not reveal this effect in any clearly recognizable form (Fig. 4).

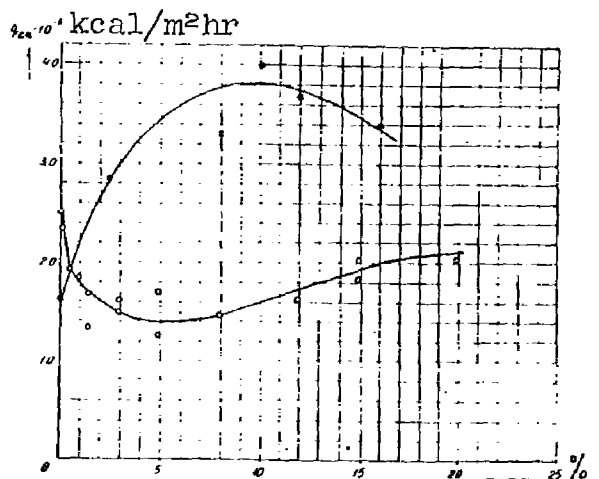


Fig. 4. The value  $q_{cr, 1}$  vs. concentration of aqueous butanol: ● - wire,  $p = 9.8$  atm (abs.); ○ - plate,  $p = 11$  atm (abs.).

#### The Effect of Forced Convection on Heat Transfer with Nucleate Boiling in Tubes

The influence of directed convection, characterized by the mass flow rate and the vapor content of the flux, on the intensity of nucleate-boiling heat transfer depends on the heat-flux density and the physical properties of the heat-transfer agent. Sterman,

Styushin, Sorokin, et al., [18, 41 - 44] have devoted much effort to this problem.

With high pressure and strong heat fluxes, the flow rate depends only slightly on the nucleate-boiling heat-transfer coefficient; this has been verified by the experiments of Alad'yev, Armand, et al., [2, 47].

At low pressure and moderate heat fluxes this effect can be quite essential.

As the author has shown [24, 29], the effect of the influence of circulation rate on the nucleate heat-transfer coefficient can be estimated from the ratio of the heat-transfer coefficients during the flow of a nonboiling liquid  $\alpha_0$  and during developed boiling  $\alpha_{00}$ .

In that region where vapor formation has no effect, we have

$$\frac{\alpha}{\alpha_0} = f\left(\frac{\alpha_{00}}{\alpha_0}\right), \quad (25)$$

whereupon

$$\left. \begin{array}{l} \frac{\alpha_{00}}{\alpha_0} = 0, \quad f = 1, \quad f' = 0; \\ \frac{\alpha_{00}}{\alpha_0} \rightarrow \infty, \quad f \rightarrow \frac{\alpha_{00}}{\alpha_0}, \quad f' \rightarrow 1. \end{array} \right\} \quad (26)$$

The simplest interpolation formula which satisfies conditions (26) is

$$\frac{\alpha}{\alpha_0} = \sqrt[n]{1 + \left(\frac{\alpha_{00}}{\alpha_0}\right)^n}. \quad (27)$$

Figure 5 shows  $\alpha$  vs.  $w_0$  and  $q$  from one of Sterman's experiments. From the figure it is clearly evident that the influence of circulation rate decreases with increasing  $q$ .

Processing of these experiments has shown that a value  $n = 2$  can be considered convenient for practical calculations [29].

The value  $\alpha_{00}$ , the heat-transfer coefficient during developed boiling, is determined for a given liquid by the formula

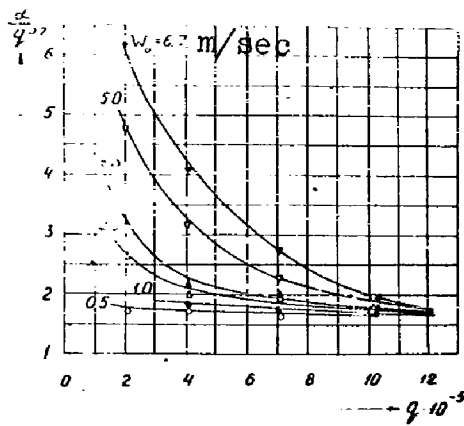


Fig. 5. The value  $\frac{\alpha}{q^{0.7}}$  vs.  $q$  and  $w_0$  for water,  $p = 2$  atm (abs.), tube diameter 16 mm.

$$\alpha_{bo} = C(p) q^{0.7}. \quad (28)$$

The values of the pressure function  $C(p)$  according to experiments with the boiling of water in tubes are given in Fig. 6 from data of Alad'yev, Sterman, et al. [2, 44, 47]. This relation is qualitatively analogous to, but somewhat lower than, that obtained by Borishanskiy [7] in his experiments with free-convection boiling in a large-volume liquid. This 20-30% quantitative discrepancy is of relatively slight significance.

This dependence holds when  $\beta < 0.7$ . It should be noted that the available experimental data are by no means ample for these vapor contents. For the region  $\beta > 0.7$ , however, the available data are insufficient for generalizations.

Experiments by Tarasova, Armand, and Kon'kov [47], conducted at a pressure of 170 atm (abs.) have shown that when  $\beta > 0.7$  there occurs first an increase and then a sharp drop in the value of the heat-transfer coefficient.

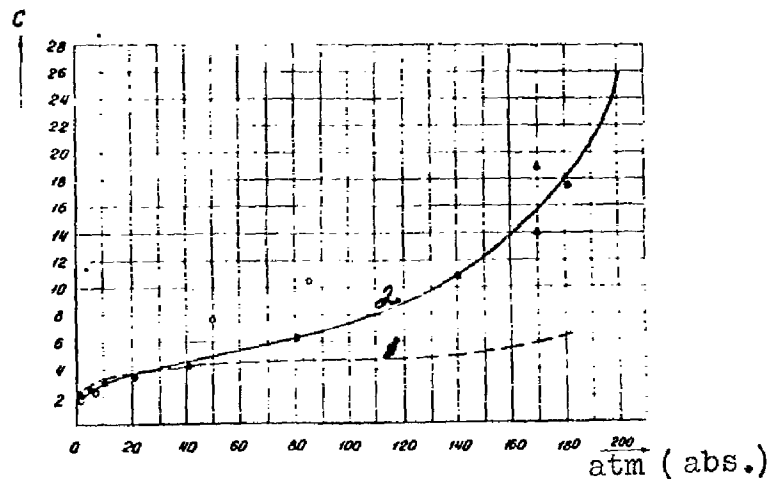


Fig. 6. The value  $C(p)$  in Formula (28) from experiments with boiling water in tubes: ● - according to [2]; ○ - according to [44]; Δ - according to [47]; curve 1 according to Kruzhillin's formula; curve 2 according to Kutateladze's formula. In the general formulas the proportionality factors are reduced by 20%.

#### Developed Nucleate-Boiling Heat Transfer

The lack of influence of velocity and channel dimensions on the heat-transfer coefficient during developed nucleate boiling (at least when  $\beta < 0.7$  and  $D > 3 \sqrt{\frac{\sigma}{\rho g}}$ ) is associated with the fact that in this case the washing of the heating surface by the liquid is completely determined strictly by the process of vapor formation. This is yet another verification of the qualitative difference in heat-transfer mechanisms during nucleate boiling and during the origination of film boiling, since the value  $q_{cr}$  is quite clearly a function of velocity in those cases where  $\alpha$  is practically independent of it.

Thus, during developed boiling, only the heat-flux density  $\underline{q}$  is an operational factor with respect to the heat-transfer coefficient. In addition, influence is exerted by the group of physical properties of the medium which change along the saturation line.



The probable distribution of vapor formation centers along the heating surface and the process of the generation of vapor bubbles cause the hydrodynamic and thermal situations to be unstable in the near-wall region, as has been mentioned previously.

In this regard, the values of the heat-transfer coefficients are far from identical in various experiments. Proper attention is not devoted to this circumstance, and individual experimenters persistently consider that only their data are reliable.

When determining the criteria of similarity of the heat-transfer process it is necessary to examine the interaction of the entire flux with the heating surface, i.e., the investigation must not be restricted merely to local conditions, as was done in the problem of the critical regime. This causes as yet insurmountable difficulties in the formulation of a corresponding system of equations to which, in addition, must be added, in the given case, conditions at the boundaries of the system.

However, these difficulties differ little from similar difficulties in the analytical formulation of ordinary turbulent flow where the flux also consists of the statistical aggregate of final formations which are variable in time and space.

Since during boiling various areas in space have differing physical properties (liquid and vapor), the mathematical formulation of the problem should encompass relationships for the phenomena within each region and also for the interactions (mechanical and thermal) on their surfaces, i.e., at the phase interfaces.

The conditions on the boundary of the entire system, i.e., on the heating surface, should be given independently.

All reactions in a two-phase flux of any complexity are a direct result of a group of local reactions. Therefore, if we assume that

this aggregate does not introduce any particular physical magnitudes, the apparatus for deriving the similarity criteria consists of a system of differential motion and thermal-conductivity equations in each phase, the conditions of local reactions at the interfaces, and the conditions on the heating surface (proper boundary conditions).

This problem has been presented in such form by the author [23, 24] and later by Eygenson [53].

This approach to the derivation of similitude conditions can be formulated in two ways:

- 1) we write the heat-transfer and motion equations in each phase in differential form, the equations of the reaction on a differential element of the interface, and the conditions at the system boundaries;

- 2) we write the heat-transfer and motion equations in differential form for the dominant phase in the process (for the liquid in nucleate boiling). The second phase is treated as the final elements of specific form (e.g., spherical vapor bubbles). We write the motion equations for the center of gravity of such an element and the heat-balance equations. The conditions at the boundaries of the system are given as in the first case.

We can also compile a system of equations by considering the new phase as a heat and mass sink [10, 42]. Methodologically, this is no different from that given above, but in this case the discrete elements of the new phase must be considered as material points. Therefore, such a method can be used when the region encompassed by the process is very large compared with the elements of the dispersed phase.

Both methods are basically identical, since the equation of

motion of the center of gravity of a bubble can be obtained by solving the combination of motion-equation systems for both phases. Such solutions have been in final form for individual regions of Reynolds numbers by Hadamard [61], Rybchinsky [69], and Levich [30].

Naturally, the system of equations should correspond to the model used for the actual physical process, and be closed, in the sense that the number of equations corresponds to the number of unknowns contained therein. However, the degree of correspondence of the model to the actual process is determined, in final analysis, by comparison with the results of generalization of the experimental data. This factor, naturally, is not a feature of the boiling process, but has general significance [19].

Examination of the motion and thermal-conductivity equations of the liquid, taking into account the equations of motion and of mechanical and thermal reactions of the vapor phase, yields three criteria:

$$\begin{aligned} Re_* &= \frac{q}{2r''} \left( \frac{\sigma}{r-r''} \right)^{1/2}; \\ Ar_* &= \frac{q}{\nu^2} \left( \frac{\sigma}{r-r''} \right)^{3/2} \left( 1 - \frac{r''}{r} \right); \\ Pr &= \frac{\nu}{Q}. \end{aligned} \tag{29}$$

Here, as the velocity scale, we have the value  $\frac{q}{r''}$ , while as the linear-dimension scale we have  $\sqrt{\sigma/\gamma\gamma''}$ .

With very intense boiling, the near-wall layer can be so agitated that the influence of molecular viscosity can be disregarded in the motion equation. In such a case, instead of the three criteria (29) we have only two:

$$\left. \begin{aligned} Pe_* &= Pr Re_* = \frac{q}{2r''Q} \left( \frac{\sigma}{r-r''} \right)^{1/2}; \\ Pr^2 Ar_* &= \frac{q}{Q^2} \left( \frac{\sigma}{r-r''} \right)^{3/2} \left( 1 - \frac{r''}{r} \right). \end{aligned} \right\} \tag{30}$$

This holds, in particular, for media with  $Pr \ll 1$ . From the boundary conditions it follows that

$$Nu_* = \frac{\alpha}{\lambda} \left( \frac{\sigma}{r-r''} \right)^{1/2}. \quad (31)$$

This criterion was introduced by Jakob [65].

The condition of continuity of the temperature field at the phase interfaces has the form

$$T_{1_1} = T_{1_2} = T_1'' \quad (32)$$

If, as was done by Kruzhilin [21], we define  $T_1''$  according to Formula (4) we get another new criterion which can be represented in the form

$$K_t = \frac{(2r'')^2}{Ac\gamma T'' \sqrt{\sigma(r-r'')}} \quad (33)$$

If we consider the kinetics of evaporation, as was done by Zysin [14, 24], we have the criterion

$$K_p = \frac{P}{\sqrt{\sigma(r-r'')}} \quad (34)$$

As was shown above, the terms of the equation that result in these two criteria are at least conjugate.

If we assume that  $\frac{dp}{dT''} \approx \frac{p}{T''}$ , we have

$$\frac{K_t}{K_p} = K_s \frac{r''}{r} \left( 1 - \frac{r''}{r} \right) \quad (35)$$

where

$$K_s = \frac{2}{cT''} \quad (36)$$

It is difficult to process experimental data with the inclusion of the criteria  $K_t$  and  $K_p$  (or  $K_p$  and  $K_s(\gamma''/\gamma)$ ). It appears, however, that for many liquids there is practically a single-value relationship between these criteria. This explains the fact that

empirical formulas constructed for a limited system of criteria of the type

$$\left. \begin{aligned} Nu_* &= f_1(Pr; Re_*; K_p) \\ Nu_* &= f_2(Pr; Re_*; K_i) \end{aligned} \right\}, \quad (37)$$

give more or less satisfactory results.

For a specific liquid, one of these processing methods has slight preference. Tobilevich [49] and certain other authors have shown that processing using the criterion  $K_p$  gives better results, particularly in the region of high pressures. In particular, this is true of water, an important working substance (Fig. 6).

In striving to improve the agreement with experimental data, certain authors have constructed formulas that contain a limited number of criteria and have individual constants for each liquid. In certain formulas these constants are even dimensional. Such formulas have been derived by Rohsenow [68], Alad'yev [2], Jakob [66], Tolubinskiy [50], and Levy [62]. Evidently, in this case simple empirical formulas for a given mass are preferable.

Alad'yev has given a successful formula of this type for water; this can be given conveniently in the form

$$\alpha = \frac{C q^{0.7}}{1 - 2.56 \cdot 10^{-11} q^4}, \quad (38)$$

where  $C$  is a constant which characterizes the heating surface. With tube boiling,  $C \approx 2.1$ ; for large-volume boiling in brass pipes  $C \approx 2.6$ ; for large-volume boiling in stainless steel pipes  $C \approx 3$ .

Methods of constructing empirical formulas on the basis of the theory of corresponding states of matter are also useful in this respect. This question has been treated by Novikov [34], Rychkov [40], Borishanskiy [8], and others.

## Conclusions

1. The available theoretical and experimental data on the mechanism of nucleate boiling pertain mainly to conditions in which the bubbles still react with one another only weakly. When a saturated liquid boils in a region of high heat-flux densities, the mechanism of the development of vapor bubbles changes substantially. Further investigations of such types are necessary to construct a relatively complete theory of nucleate boiling.

2. The hypothesis of the hydrodynamic nature of crises in boiling and the development of a crisis from local disturbances in the stability of the two-phase boundary layer has made it possible to formulate a system of differential equations to describe this process, to select a rational form of the specific criterion, and select a system of basic specific criteria.

On the basis of this hypothesis, theoretical formulas have been derived for the first critical heat-flux density in a large volume of free-convection liquid.

The qualitative conclusions from the hydrodynamic hypothesis are also verified by data on forced flow.

Further development of this problem requires vast study of the critical phenomena for the flow of liquids with various physical properties and accurate notation of the experiment conditions.

It is particularly important to gather experimental data on the positive vapor contents of a flux.

3. The influence of circulation rate on the nucleate-boiling heat-transfer coefficient depends on the relation of intensities of heat transfer during ordinary convection and during developed

boiling. In certain cases this influence can be quite substantial and aid in intensifying the operation of the heat-exchanger.

4. Strict formulation of the problem of the intensity of nucleate-boiling heat-transfer is possible only by examining statistically the multitude of final elements which are variable in space and time. Such statistics are lacking at the present time.

The above-mentioned methods for circumventing this difficulty in deriving similitude conditions result in a relatively complete system of hydrodynamic and thermal criteria. Conditions on the heating surface cannot be strictly formulated.

5. Practically no studies have been made of conditions of heat-transfer during the boiling of binary mixtures and for certain other special cases. The amount of work done along this line in no way corresponds to the practical and theoretical significance of the problem.

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