Solving Quadratic Equations Review Sheet

METHODS FOR SOLVING QUADRATICS

When the equation is in **standard form** $ax^2 + bx + c = 0$ there are several ways to proceed to get the solution.

FACTORING: Always look for GCF first and make the equation easier to work with. Then decide if there is a difference of squares or perfect square trinomial to deal.

$2x^2 - 8 = 0 \rightarrow\ 2(x^2 - 4) = 0 \rightarrow\ 2(x-2)(x+2) = 0 \rightarrow\ \ x = 2\ and\ x = -2$
$4x^2 - 14x - 30 = 0 \rightarrow \div 2\ \rightarrow\ \ 2x^2 - 7x - 15 = 0 \rightarrow\ (2x+3)(x-5) = 0\ \rightarrow\ \ x = 5\ and\ x = -\frac{3}{2}$

SQUARE ROOT PRINCIPLE: If the equation has a single squared term, the square root principle is a good method to use.

	STEPS
$2x^2 - 7 = 3$	1. Isolate the squared term.
$x^2 = 5$	2. Square root both side and remember $\pm$
$x = \pm\sqrt{5}$	3. Reduce radical if possible.

$(2x-3)^2 - 4 = 12$
$(2x-3)^2 = 16$
$2x - 3 = \pm 4$
$2x = 3 \pm 4$
$x = \frac{3 \pm 4}{2}$
$x = \frac{7}{2}\ \ and\ \ x = -\frac{1}{2}$

Note: If you expand you get $4x^2 - 12x + 9 - 4 = 12$
$4x^2 - 12x - 7 = 0$
$(2x+1)(2x-7) = 0$
$x = \frac{7}{2}\ \ and\ \ x = -\frac{1}{2}$

COMPLETING THE SQUARE: If the equation cannot be factored, then use this approach.

	STEPS
$2x^2 - 12x - 10 = 0$	1. Divide each term by "a" and isolate the constant term.
$x^2 - 6x + \quad = 5 +$	2. Multiply "b" by ½ and square → $\left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{6}{2}\right)^2 = 9$
$x^2 - 6x + 9 = 5 + 9$	3. Add this number to both sides.
$(x-3)^2 = 14$	4. Square root both sides (remember $\pm$)
$x - 3 = \pm\sqrt{14}$	5. Solve and reduce completely.
$x = 3 \pm \sqrt{14}$	

The next one is a beast of a question.

$5x^2 - 3x - 4 = 0$

$x^2 - \frac{3}{5}x + \frac{9}{100} = \frac{4}{5} + \frac{9}{100}$

$\left(x - \frac{3}{10}\right)^2 = \frac{89}{100}$

$x - \frac{3}{10} = \frac{\pm\sqrt{89}}{10} \rightarrow x = \frac{3\pm\sqrt{89}}{10}$

QUADRATIC FORMULA: Although completing the square will solve any question, sometimes the numbers can be rather large and may be too challenging to work with.

$221x^2 + 59x - 120 = 0$ Hmmm…. Not so sure I want to divide every term by 221.

$a = 221 \quad b = 59 \quad c = -120$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \rightarrow x = \frac{-59\pm\sqrt{59^2-4(221)(-120)}}{2(221)} = \frac{-59\pm\sqrt{3481+106080}}{442} = \frac{-59\pm\sqrt{109561}}{442} = \frac{-59\pm331}{442}$

$x = \frac{272}{442} = \frac{8}{13} \quad and \quad x = -\frac{390}{442} = -\frac{15}{17}$

If you are feeling supppper nerdy…. You could try to factor this one.

$(13x - 8)(17x + 15) = 0$

General Note: It is always a good idea to try factoring first before you jump in to completing the square. If the equation is in general form ($ax^2 + bx + c = 0$), here are some suggestions for specific cases:

$c = 0$ **FACTOR** $x^2 - 5x = 0 \rightarrow x(x - 5) = 0 \rightarrow x = 0 \; and \; x = 5$

$b = 0$ **SQRTPrinc** $x^2 - 8 = 0 \rightarrow x^2 = 8 \rightarrow x = \pm 2\sqrt{2}$

Nature of the roots: When solving a quadratic equation you may get the square root of a negative number. If the radicand is negative for an even index radical it is important to understand what this means. This condition means there is no real number solution to the question. However it is not fair to say there is no answer. In fact, you got the answer, so there is an answer. It is just not Real.

The Discriminant: This can be used to determine the nature of the roots. Three conditions can happen.

$b^2 - 4ac > 0$ Two real roots. NOTE: Roots mean answers in this context.

$b^2 - 4ac = 0$ One real root.

$b^2 - 4ac < 0$ No real roots. (Two imaginary roots.)

$x^2 + 9 = 0 \rightarrow x^2 = -9 \rightarrow x = \pm\sqrt{-9}$ No Real Roots or $x = \pm 3i$ (Two imaginary roots.)

Discriminant is $0^2 - 4(1)(9) = -36$ so it is negative and there is no real roots.

$2x^2 + 3x + 7 = 0$ The discriminant is $3^2 - 4(2)(7) < 0$ No Real Roots

RESTRICTIONS: If the question is a rational expressions then there will be restrictions on the denominator. If there are even index radicals then there will be restrictions on the radicand.

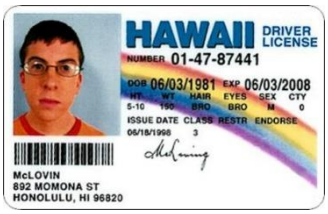
$\frac{x}{x-5} - \frac{3}{x+1} = \frac{30}{x^2-4x-5}$ → If you solve this the roots are $x = 5$ *and* $x = -3$.
Note: $x \neq 5$ *and* $x \neq -1$ This means $x = 5$ is an **extraneous root.** The only real root is -3.
$\sqrt{x+1} = x - 1$ → If you solve this the roots are $x = 0$ *and* $x = 3$.
Note: $x \geq -1$ Both answers meet this. However, $x = 0$ is an **extraneous root.** The only real root is 3.

WORD PROBLEMS

All word problems are to be answered must include:
a. Let statement b. A quadratic equation c. Solving the quadratic equation
d. A written statement of the solution.

Unless stated, all answers are to be in simplified rational form.

1. McLovin has a forged drivers license. The license is a rectangle where the length is 3 cm less than twice the width. The area of the rectangle $35\ cm^2$. Determine the perimeter of the drivers license.

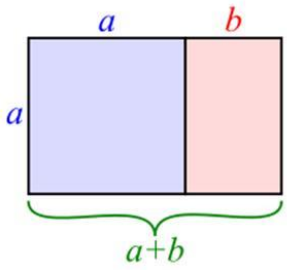


2. Jania Proudmore has a portrait of Uther Lightbringer. The portrait is a rectangular painting that measures 4 ft by 5 ft. Jania has decided to place a wooden border around the painting. The border is of equal width. The area of the wooden border is $3\ ft^2$. Determine the width of the border.

3. A rectangular sundeck is 10 ft by 15 ft. If the shorter side is increase by the same amount as the longer side is decreased, the new area is $156\ ft^2$. Determine the new dimensions of the new sundeck.

4. A rectangular yard has an area of $1250\ m^2$. The yard is divided into 4 different pens. To enclose the yard and the 4 pens, $300\ m$ of chain-link fence is used. Determine the width and length of the rectangular yard.

5. You have heard about pi ($\pi \approx 3.14$ ...). Another special number is phi. To some this is the most beautiful number in existence. It is referred to as the Golden Ratio. The value can be determine by $\frac{a+b}{a} = \frac{a}{b}$.
If $b = 1$, determine the exact value of a. You have determine the value of phi ϕ. What is the approximate value of phi to 3 decimal places.



6. Li-Ming and Sylvanas were in a bicycle relay. Li-Ming rode her bicycle for 75 km and tagged Sylvanas who rode 60 km. Sylvanas rode 5 km/h faster than Li-Ming. If Sylvanas was 1 hour faster than Li-Ming, determine the average speed of Li-Ming.
7. Charon's boat can travel at 10 km/h. When his boat goes up the river Styx (24 km) he is 1 hour slower than when he rides back down the river. How fast is the speed of the current?

8. 

In order to be a king, it is important to know a great many things. Some of which may seem trivial at the time but can turn out to be life-saving bits of information. In his quest to find the Holy Grail, Arthur (King of the Briton's) is asked by the bridge keeper to answer three questions before he can cross the Bridge of Death. The last question is to determine the air speed velocity of an unladen swallow. There is of course African and European swallows.
A European unladen swallow flying from Camelot to Chesterville encounters a head wind of 4 mph (miles per hour). Once it knows the price of coconuts, it returns to Camelot but is pushed along by a tail wind of 4 mph. The total flying time there and back is half an hour. The distance between Camelot and Chesterville is 5 miles. Determine the average flying speed of an unladen European swallow. Round answer to 2 decimal places.

Answers:

1. 24 cm **2.** $\frac{-9+\sqrt{93}}{4}\ ft$ **3. 12 ft X 13 ft**
4. 10m X 125 m or 50 m X 25m
5. $\frac{1+\sqrt{5}}{2}$ **approximately 1.618**
6. 25 km/h **7. 2 km/h** **8. 20.77 mph**